MATHEMATICAL LEARNING OPPORTUNITIES IN PROSPECTIVE ELEMENTARY TEACHERS' RESPONSES TO A COMPARING FRACTION PAIR TASK

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We identified five distinct mathematical learning opportunities in 26 prospective elementary teachers' (PTs) written responses to comparing a fraction pair. If their instructor were to take advantage of these learning opportunities by engaging the class in making sense of these PT's contributions when comparing fractions, the class could move toward better understanding (a) the use of a benchmark, (b) the need to consider both the size of the pieces (denominator) and the number of pieces (numerator), (c) the importance of using the same size whole, (d) the set model, and (e) the use of equivalent fractions. We also discuss how our framework shifts the focus of using the PTs' responses from a deficit perspective to a strength one—using student contributions as resources to develop the mathematical learning of the class.

Keywords: Mathematical Knowledge for Teaching, Rational Numbers, Preservice Teacher Education, Reasoning and Proof

Borasi (1994) found that engaging learners in examining their errors provides learning opportunities that can develop students' understanding of mathematics. However, little is known about what important mathematics each opportunity offers and whether other types of students' responses, such as a correct response that students may not yet recognize as correct, could be used as sources of learning mathematics. To build on Borasi's study, we focused on investigating learning opportunities across the full range of students' mathematical responses. We aimed to "interpret [students' responses] as resources or assets, and build upon them," rather than "interpret them as flawed understanding, and correct them…or interpret them as a lack of understanding, and supplement them" (Scheiner, 2023, p. 52).

We focused on investigating students' responses to fraction comparisons because comparing fractions has been found to be a challenging topic for not only K-12 students (e.g., Clarke & Roche, 2009) but also prospective teachers (e.g., Liu, 2021) and teachers (e.g., Sullivan et al., 2009). We respond to Liu's (2021) call for more studies investigating ways to develop prospective teachers' fraction proficiency, including fraction comparison, by examining this research question: *What mathematical learning opportunities emerge in prospective elementary teachers' responses to a comparing fraction pair task?*

Theoretical Framework

Our identification of mathematical learning opportunities was informed by the *mathematical point* construct introduced in the conceptualization of a Mathematically Significant Pedagogical Opportunity to Build on Student Thinking (MOST; Leatham et al. 2015; Van Zoest et al., 2016). A MOST is a student contribution (e.g., a spoken idea or a piece of student work) that provides an opportunity to "engage the class in joint sense making about the contribution to better understand the important mathematics within it" (Van Zoest et al., 2023, p. 244). This important mathematics is the *mathematical point* of the contribution. Identifying the mathematical point of a student contribution is essential to determining whether it is a MOST and could be effectively built on (see Leatham et al., 2022). The mathematical point is also foundational for facilitating a discussion about a MOST—turning the contribution over to the class so they can grapple with it productively, conducting the conversation around the MOST, and identifying the mathematics to be made explicit at the end of the discussion (Leatham et al., 2022). Unlike Leatham and colleagues (2015), who focused on student contributions that occurred *during* a whole-class discussion, in the study we report here, we focused on the mathematical learning opportunities provided in written contributions immediately *before* the whole-class discussion to explore the full spectrum of opportunities available for the teacher to build on. Our definition of a *mathematical point of a student contribution* is the well-specified statement of a mathematical truth that: (a) the class could move toward understanding if they were to engage in joint sense-making of the contribution; and (b) is most closely related to the student mathematics of the contribution. In this study, the student contributions were prospective teachers' (PTs') written responses to a comparing fraction pair task. We analyzed these responses to identify the opportunities they would provide for PTs' mathematical learning if their instructor were to engage the whole class in jointly making sense of the contribution.

Methodology

This study is part of a larger study investigating mathematical struggle during a whole-class discussion of a MOST (as defined by Leatham et al., 2015). The participants were 26 PTs enrolled in a course focused on developing children's abilities to reason numerically. We collected the PTs' responses to our focal task comparing a fraction pair: "Use words and/or pictures to justify your thoughts about: Which is Bigger? $\frac{6}{7}$ or $\frac{7}{8}$?" This task seemed optimal for our study for several reasons: (a) the course instructors identified it as a task that PTs typically struggle with, (b) comparing fractions is a topic the PTs likely will be required to teach in their future courses (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), and (c) comparing two fractions whose numerator is one less than its denominator has been identified to be particularly challenging for learners (e.g., Clarke & Roche, 2009).

The unit of analysis of this study is a *contribution*—a solution to the focal task. Typically, each PT's written response to the task was one contribution. However, a few PTs solved the focal task in more than one way. In this case, each distinct solution was considered a separate contribution. The 26 PT responses resulted in 31 contributions. Our iterative analysis approach included *open coding*, *analytic memoing*, *analytic coding*, and *negative case analysis* (Merriam & Tisdell, 2016). The first author did the initial coding to determine the mathematical point (MP) of each contribution. Then, the second author provided constructive feedback on the initial set of MPs. Both authors reconciled and refined the MPs by using the first author's analytic memoing and employing negative case analysis during the reconciling process. We brought our refined MPs to a group of mathematics education faculty and graduate students for constructive feedback. We then further refined the MPs, individually recoded the contributions, and discussed any differences in our assignment of MPs to the contributions until agreement was reached.

Results

Five distinct mathematical learning opportunities emerged in PTs' responses to comparing the fraction pair, $\frac{6}{7}$ and $\frac{7}{8}$. Each learning opportunity corresponds to a set of PT contributions that share the same mathematical point (MP). The first mathematical learning opportunity occurred in the three contributions where PTs used the whole number one as a benchmark to help compare the two fractions (see Figure 1). If their instructor were to engage the class in making sense of one of these contributions, the PTs would have the opportunity to move toward understanding

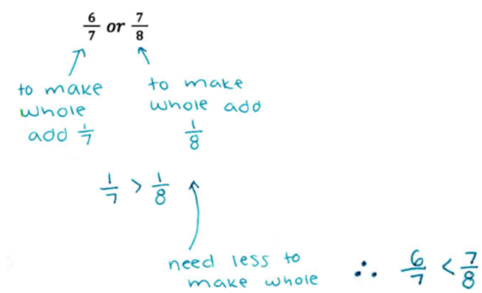
MP1: *When comparing fractions that are missing the same number of pieces to a larger benchmark, the smaller the size of each piece, the larger the fraction.*

Figure 1 illustrates a variety of contributions can have the same MP. Contribution 1 provided a clear explanation for the correct answer of $\frac{7}{8}$ being greater than $\frac{6}{7}$. Contribution 2 showed a correct visual solution with an explanation that required minor inferences [shown in brackets]. Contribution 3 illustrated the incorrect answer that $\frac{6}{7}$ is larger than $\frac{7}{8}$, with correct reasoning about comparing the size of the missing pieces of the two fractions to the whole one. Whether correct or incorrect, visual or based on words, these contributions all provided fodder for joint sense-making that could lead toward better understanding MP1.

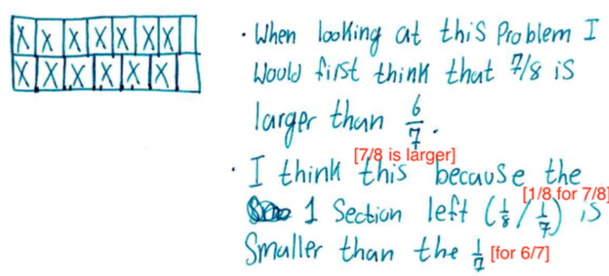
Figure 1: The Prospective Elementary Teacher Contributions that Generated the Learning Opportunity for the Class to Better Understand MP1

MP1: *When comparing fractions that are missing the same number of pieces to a larger benchmark, the smaller the size of each piece, the larger the fraction.*

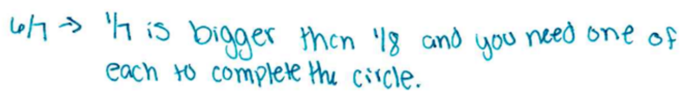
Contribution 1



Contribution 2



Contribution 3



The second mathematical learning opportunity emerged in the three contributions that depended on the meaning of the denominator and numerator as the size of the pieces and the number of pieces, respectively. This opportunity could move the PTs toward better understanding MP2: *Both the size of the pieces and the number of pieces (denominator and numerator) affect the relative size of fractions.* The third and the fourth mathematical learning opportunities both involved visual representations—rectangles, circles, mixed visuals, and set models—that required the same size whole. The twelve contributions in the third set provided the opportunity to focus on the need to start with the same size whole and accurately draw the pieces, leading to MP3: *Comparing fractions using an area model requires that the wholes are the same shape and area and that the pieces are drawn precisely*. The two contributions in the fourth set required making sense of what counts as a whole when using a set model. Engaging in joint sense-making about these contributions could provide the opportunity to move the class toward better understanding MP4: *Comparing fractions using a set model requires that each whole has the same number of objects*. The fifth learning opportunity arose when the PTs relied on a procedure for developing common denominators as their justification. The five contributions in this set could provide the opportunity for the class to engage in joint sense-

making about why these procedures work and move them toward better understanding
MP5: *When comparing two fractions, multiplying each fraction's numerator and denominator by the denominator of the other fraction (equivalent to multiplying by 1) will give both fractions a common denominator (same size pieces) so the one with the largest numerator (most pieces) will be the largest fraction*. The remaining six contributions did not have a mathematical point.

Discussion

First, accuracy does not predict whether instructors can use PTs' contributions as resources to create a mathematical opportunity to help the class better understand important mathematics emerging from those contributions. From Scheiner's (2023) study, we can imagine that if an instructor used a deficit-based orientation to inspect the three PT contributions in Figure 1, they would interpret Contribution 3 as an indicator of the PT's lack of understanding in comparing the two fractions and focus on correcting their error. In contrast, through our strengths-based lens, we recognized the contribution as an instructional resource to support the learning of the class. We were aided in this recognition by attending to the mathematical point of a student contribution. This suggests that the mathematical point of a student contribution could be used as a framework to help shift from deficit-based orientations that focus on identifying and correcting errors in PTs' work to strength-based orientations that use students' mathematical contributions as resources to develop the mathematical learning of the class. Second, it was clear in our data that the PTs were still in the process of making sense of fractions. For instance, Contribution 3 in Figure 1 represents contributions that were on the cusp of understanding (in this case, the PT expressed the reasoning for the correct answer, but chose the other option). Each of the five learning opportunities that emerged from the PTs' contributions was relevant to the rich understanding of comparing fraction pairs that they will need to effectively support their future students' learning. Finally, taking advantage of these opportunities will require teacher educators to resist the temptation to "cover" the wide range of topics that PTs will be responsible for teaching and instead to trust in the power of engaging PTs in joint sense-making about their peers' contributions. Analyses of the mathematical opportunities available in preservice teacher contributions, such as was reported in this paper, can provide important fodder for supporting preservice teachers to develop *mathematical knowledge for teaching* (Ball et al., 2008) and help them to implement Smith and Stein's (2018) practices for supporting whole-class discussion.

Conclusion

This study revealed that engaging prospective elementary teachers in solving the same comparing fraction pair task led to diverse mathematical contributions that could be used as resources to create meaningful mathematical learning opportunities for the class. Rather than looking at student work as a way to identify and correct their errors, teacher educators should both explicitly educate prospective teachers about and model student contributions as instructional resources. Future research could extend our methodology to other topic areas or investigate how teachers respond to different learning opportunities created by student contributions and the mathematics that the class learns when they are supported to engage in joint sense-making of their peers' contributions.

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