A PRELIMINARY SYSTEMATIC REVIEW ON HOW PRODUCTIVE STRUGGLE IS DEFINED IN MATHEMATICS EDUCATION RESEARCH

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This systematic review investigated how productive struggle *was defined in studies investigating productive struggle in mathematics learning. Following PRISMA guidelines, we identified 10 such peer-reviewed journal articles from the Scopus database from 2007 to 2023. We reported (a) (proxy) definitions of productive struggle for each study; (b) structural elements across the definitions—subject, action, object, and aim; and (c) synthesizing aspects across the definitions—definition foci and features of the objects. Finally, we initiated the process of rethinking together how to investigate what it means for mathematics learners to engage in productive struggle by sharpening the productive struggle construct.*

INTRODUCTION

Productive struggle has become a popular phrase in mathematics education in the United States. The first use of "productive struggle" in the Scopus database related to mathematics was by Warshauer in 2015 and by the end of 2023 it was mentioned in 274 documents. This increasing use may be in part because "support[ing] productive struggle in learning mathematics" is one of the US National Council of Teachers of Mathematics' eight mathematical teaching practices ([NCTM], 2014, p .10). NCTM (2014) cited three research documents to support their claims about productive struggle: Hiebert and Grouws' (2007) handbook chapter on teaching and learning, Kapur's (2010) study of productive failure, and Warshauer's (2011) dissertation on productive struggle. Warshauer (2011) is the only of these three documents that reports on a study about productive struggle. Kapur (2010) made no mention of *productive struggle* (although the construct of *productive failure* is closely related) and Hiebert and Grouws (2007) identified *struggle with important mathematics* as a feature of teaching that research has found to support students' conceptual understanding. Hiebert and Grouws explicitly defined *struggle* to mean "students expend effort to make sense of mathematics, to figure something out that is not immediately apparent" (p. 387). Warshauer (2011) quoted Hiebert and Grouws (2007)'s definition of *struggle* when defining her use of *productive struggle*.

Kaiser and Schukajlow (2023) called for more literature reviews in mathematics education. Given the need for researchers to investigate *productive struggle* and the lack of an established definition of what is meant by the term, this seemed a useful time to complete a systematic literature review on the definitions of productive struggle currently in use. Thus we report here on a systematic literature review of the question,

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How is productive struggle defined in studies investigating productive struggle in mathematics learning?

METHODS

Our study is a systematic literature review—a study that has "systematic and rigorous search procedures" (Kaiser & Schukajlow, 2023, p. 2). We drew on recently published recommendations for completing literature reviews (e.g., Kaiser & Schukajlaw, 2023) and followed the PRISMA (2020) checklist and flowchart.

We had two major inclusion and exclusion criteria for our review: (a) we were interested in research studies that investigated productive struggle in mathematics learning, rather than simply mentioning the term; and (b) we wanted to focus on rigorous studies. As have other mathematics education researchers (e.g., Nieminen et al., 2023; Phan et al., 2022), we started with the Scopus database because it "is the largest abstract and citation database of peer-reviewed literature," covering over 25,000 journals across all disciplines (Scopus blog, 2023, homepage). After our initial comparison with the ERIC and Education Source databases revealed no additional peer-reviewed journal articles that met our criteria, we limited our search to Scopus for this preliminary systematic review. To meet our first criterion, we searched the article title, abstract, and keywords for any mention of "productive struggle" or mathematics (using "math*" to capture variations of the term). To meet our second criterion, we used publication in peer-reviewed journals as a proxy for "rigorous." Because Hiebert and Grouws' (2007) article is often credited with drawing the field's attention to the idea of struggle in mathematics education, we used 2007 as the starting point for our search. We included articles that had a publication date through the end of 2023. Figure 1 illustrates our process of identifying studies and the numbers that resulted at each step. Our initial search identified 30 articles. We first screened these 30 articles by reading their title, abstract, and keywords carefully to decide whether the research investigated productive struggle in mathematics learning. We excluded a total of 18 articles: (a) two because they were suggestions for practitioners rather than reports of research studies; (b) three because they focused solely on investigating beliefs or attitudes; and (c) thirteen because they did not investigate productive struggle in mathematics learning (e.g., investigating teachers' productive struggle when learning to design a mathematics curriculum, rather than when learning mathematics). We then considered the full text of the 12 remaining studies to verify that they did investigate productive struggle in mathematics learning. Two studies did not meet this criterion and thus were excluded from our data set. The remaining 10 studies were included in our review.

Our analysis included: (a) identifying a definition of productive struggle used in each study; (b) deconstructing the structural elements of each definition of productive struggle; and (c) synthesizing across those structural elements. For (a), we first searched each article for an explicit mention of the definition of productive struggle used in the study. We considered it an *explicit definition* if there was a clearly stated

definition and it was clearly stated that it was the definition used in the study. If a definition was clearly stated, but it was not clearly stated that it was the definition used in the study, we searched the article for confirming and disconfirming evidence of its use. If there was no disconfirming evidence, we identified the definition as *inferred explicit*. If no, or multiple, clearly stated definitions were found, we searched the article for passages that provided evidence about the definition of productive struggle that might have been used in the study. We then looked across these collected passages for commonalities and selected the passage that best captured those commonalities.

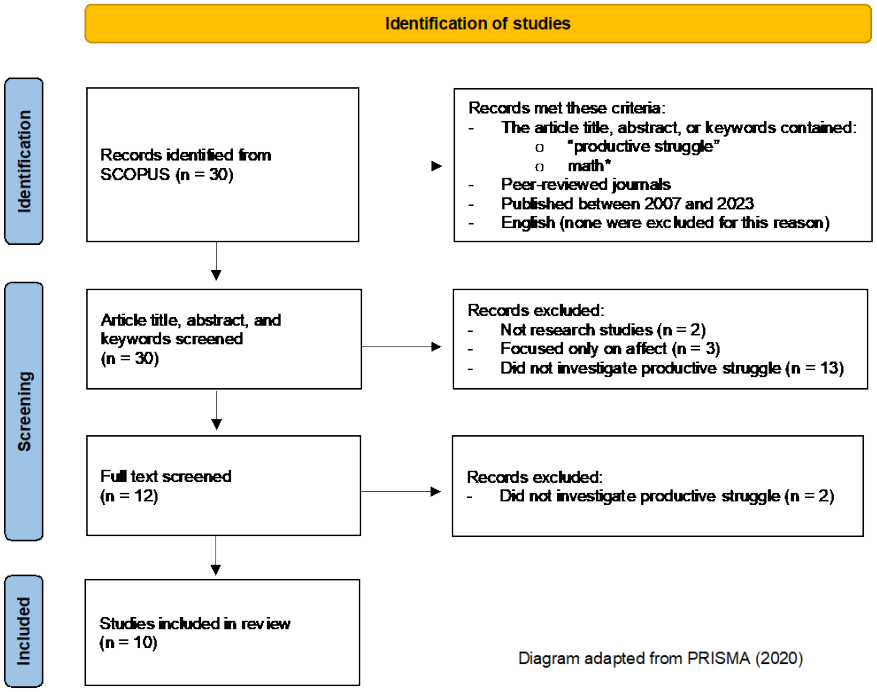

Figure 1: Process of identifying studies included in the systematic review.

This selected passage served as a proxy for the study's definition of productive struggle and was identified as *inferred*. See Figure 2 for a summary of the codes and their definitions.

Code Name	Code Definition
Explicit	The authors stated clearly what definition of productive struggle they used in their study.
Inferred Explicit	The authors cited an existing definition that appeared to be the definition of productive struggle they used in their study, but did not state that it was.
Inferred	The authors (1) cited multiple existing definitions without indicating which one they used; or (2) only gave descriptions that provided insight into how they might be defining productive struggle in their study.

Figure 2: Code names and definitions for analyzing productive struggle definitions

To increase the trustworthiness of our review, a mathematics education graduate student researcher familiar with productive struggle double-coded the data for eight of the articles (80%) with the first author. After individually coding, this researcher and the first author compared their codes and through discussion identified the passages that best captured the definition of productive struggle being used in the article. The two authors reviewed the results, revisiting the articles when any questions arose, and

agreed on a (proxy) definition for each study, which was reviewed and confirmed by the third researcher.

For (b), the part of our analysis where we deconstructed the structural elements of each definition of productive struggle, the unit of analysis shifted from the study to the identified (proxy) definition. The first author analyzed the (proxy) definitions and identified structural elements that appeared within them. The second author made minor adjustments to this deconstruction and the third researcher verified the accuracy of the results. Finally, for (c) the first author synthesized the structural elements of the 10 productive struggle (proxy) definitions to develop themes across them. The second author and third researcher checked the validity of the synthesis arguments and the strength of the supporting evidence the first author provided.

RESULTS & DISCUSSION

Figure 3 shows the (proxy) definitions of productive struggle identified in the 10 studies included in our systematic review. Although we were able to identify a (proxy) definition for each study, only 30% of the articles in our study included explicit definitions of productive struggle (W15, W21, W23).

Authors (ID)	Year	(Proxy) Definition of Productive Struggle	Page	Code
Warshauer (W15)	2015	"By students' productive struggles, I refer to a student's 'effort to make sense of mathematics, to figure something out that is not immediately apparent' (Hiebert [&] Grouws, 2007, p. [387])"	376	Explicit
Warshauer et al. (W21)	2021	"By productive struggle, we mean what occurs when 'students expend effort in order to make sense of mathematics, to figure out something that is not immediately apparent' (Hiebert [&] Grouws, 2007, p. 387)"	89-90	Explicit
Warshauer et al. (W23)	2023	"By productive struggle, we refer to Hiebert and Grouws' conceptualization (2007) that 'students expend effort in order to make sense of mathematics, to figure out something that is not immediately apparent' (p. 387)"	3	Explicit
DiNapoli and Miller (D22)	2022	"…, Hiebert and Grouws (2007) defined productive struggle as 'effort to make sense of mathematics, to figure something out that is not immediately apparent' (p. [387])"	2	Inferred Explicit
Zeybek (Z16)	2016	"Hiebert and Grouws (2007) [defined] struggle as an intellectual effort students expend to make sense of mathematical concepts that are challenging but fall within the students' reasonable capabilities"	396	Inferred Explicit
Aljarrah and Towers (A22)	2022	"...iterative cycles of 'expressing, testing, and revising mathematical interpretation–and of sorting out, integrating, modifying, revising or refining clusters of mathematical concepts from various topics within and beyond mathematics' ([Lesh & Zawojewski, 2007,] p. 782)"	857	Inferred
English et al. (E23)	2023	"We identified a struggle as resolving productively if it leads to the student's (a) reflection on the limits of his or her previously established knowledge and ability (English, 2013) and (b) perseverance with the activity towards understanding while remaining cognitively engaged in a challenging task (Warshauer, 2015)"	5	Inferred
Granberg (G16)	2016	"A successful, productive struggle would result in the restructuring of mental connections in more powerful, useful ways through which the problem at hand would make sense and new information, ideas and facts would become assimilated (Hiebert & Grouws, 2007)"	34	Inferred
Rahman (R23)	2023	"Productive struggle is when students persevere through challenging tasks leading to mathematical understandings (VanLehn et al., 2019[*])"	113	Inferred
VanLehn et al. (V21)	2021	"...productive struggle [is when students] work hard…to solve challenging, open-ended problems that afford many mathematical insights and discussions"	994	Inferred

*Note that the VanLehn et al.'s (2019) article cited here is an online first version of VanLehn et al.'s (2021) article.

Figure 3: (Proxy) definitions of productive struggle in math education research

The lack of an explicit definition leaves open the possibility of misinterpretation, makes it difficult for researchers to build on each other's work, and inhibits progress in the field. Note that 80% of the articles in our study cited Hiebert and Grouws' (2007) *struggle* definition (all except A22, V21) and all three studies with an explicit definition (W15, W21, W23) used Hiebert and Grouws' (2007) definition for *struggle* as their definition for *productive struggle*. In addition, both of the inferred explicit proxy definitions referred to Hiebert and Grouws' (2007) *struggle* definition. This use blurs the line between *struggle* and *productive struggle*. While all of these articles include descriptions of productive vs. unproductive struggles, it is worth thinking about whether that difference should be clearly articulated by explicitly defining what is struggle, what is productive struggle, and what makes a struggle productive. Doing so would help minimize the possibility of misinterpretation and make it easier for researchers investigating productive struggle in mathematics learning to build on each other's work.

Our deconstruction of the structures within each (proxy) definition of productive struggle revealed four distinct structural elements: (a) the **subject**, (b) the **action**, (c) the **object** the subject needs to do that action, and (d) the **aim** of that action. Figure 4 shows the structural elements for each definition in our study.

ID	Subject	Action	Object	Aim
W15	student's	effort	mathematics, something that is not immediately apparent	to make sense of, to figure…out
W21	students	expend effort	mathematics, something that is not immediately apparent	in order to make sense of, to figure out
W23	students	expend effort	mathematics, something that is not immediately apparent	in order to make sense of, to figure out
D22	-	effort	mathematics, something that is not immediately apparent	to make sense of, to figure…out
Z16	students	expend [effort]	mathematical concepts that are challenging but fall within the students' reasonable capabilities	to make sense of
A22	-	expressing, testing, and revising…sorting out, integrating, modifying, revising or refining	mathematical interpretation…clusters of mathematical concepts from various topics within and beyond mathematics	-
E23	student's	reflection, perseverance	on the limits of his or her previously established knowledge and ability, the activity… a challenging task	towards understanding
G16	-	reconstructing	mental connections	[the problem] would make sense and new information, ideas and facts would become assimilated
R23	students	persevere	challenging tasks	leading to mathematical understandings
V21	students	work hard	challenging open-ended problems	to solve…afford many mathematical insights and discussions

Figure 4: A deconstruction of the structural elements of each (proxy) definition of productive struggle

We share here two observations from our synthesis of the information in Figure 4: differences in the definition foci and notable features of the objects. First, we noticed a difference between the (proxy) definitions that had the action of "effort" and the aim "to make sense of" (W15, W21, W23, D22, Z16) and the proxy definition for G16, which gave a more specific action of "reconstructing" and suggested that this action would lead to sense being made and new information, ideas and facts becoming assimilated. This led us to wonder about the advantages of having a broad definition of productive struggle versus specifying actions that students are expected to do when engaging in productive struggle, as did G16 and A22. We also wondered about what exactly made a struggle productive: the opportunity to better understand a mathematical idea or better understanding that idea. We can see advantages either way, but it seems important to state clearly which approach a study has taken. Second, we noticed two notable features of the objects: challenging and within reach. We interpreted "something that is not immediately apparent" (W15, W21, W23, D22) as synonymous with "challenging" (Z16, E23, R23, V24). Thus, 80% of the articles in our study had challenge as a part of their (proxy) definition of productive struggle. In contrast, only one article specified that the objects (e.g., task, mathematical concept) should "fall within the students' reasonable capabilities" (Z16). Given that Hiebert and Grouws (2007) described Vygotsky's (1978) zone of proximal development as "the space within which a student's struggle is likely to be productive" (p. 388), we wondered whether the object of the struggle "being within reach" is just as important to a definition of productive struggle as the object being challenging.

CONCLUSION

To enable the field to rethink together how we can investigate what it means for mathematics learners to engage in productive struggle, we shared our preliminary systematic literature review about how *productive struggle* is defined in mathematics education research. Our findings provided insight into the explicitness, coherence, and variation of current definitions. They also highlighted the importance of all researchers explicitly identifying the key terms they use in their work. Doing so would support moving the field forward as it would increase the ability of researchers to accurately build on each other's work and clarify their contributions to the field.

We limited our research to studies investigating productive struggle in mathematics learning published in peer-reviewed journal articles included in the Scopus database. Doing so potentially excluded valuable insights. For example, Kapur's definition of *productive failure* as "the design of conditions for learners to persist in generating and exploring representations and solution methods (RSMs) for solving complex, novel problems" (Kapur, 2021) seems directly related to productive struggle. Similarly, Sengupta-Irving and Agarwal's (2017) study was excluded from our study because it focused on perseverance rather than productive struggle. However, they explicitly defined productive struggle using Hiebert and Grouws' (2007) definition of struggle

as part of their explicit definition of *perseverance in problem solving as collective enterprise.* This idea of "collective enterprise" seems worth considering as the (proxy) definitions in our study did not directly address the potentially collective nature of productive struggle (see, for example, Kamlue & Van Zoest, 2022). These two examples illustrate why it might be useful to expand our literature review. An expanded systematic literature review could include (a) aspects of research on productive struggle that we excluded, (b) other types of publications (e.g., dissertations, book chapters, and conference proceedings), and (c) other fields that are related to mathematics education. The first and second expansions would provide a more comprehensive look at how productive struggle is defined in mathematics education research, and the third would provide insight into how those in other fields are using the term. Future research could use our methodology and findings to sharpen the construct of productive struggle for the field of mathematics education.

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