A PRELIMINARY LOOK AT PROSPECTIVE MATHEMATICS TEACHERS' PRODUCTIVE STRUGGLE

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This exploratory study aims to describe what productive struggle looks like when prospective mathematics teachers in a middle school mathematics methods course engage with a challenging mathematics task. We hypothesize that a productive struggle consists of different types of struggles—goal struggle, strategy struggle, and sub-strategy struggle—that can coexist simultaneously. We provide insight into the complexity of prospective teacher productive struggle and how it differs from that of middle school students. This information is useful for teacher educators who want to capitalize on the opportunity productive struggle offers for prospective teacher learning.

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In general, people perceive struggles as things they should avoid. However, the National Council of Teachers of Mathematics (NCTM; 2014) views students' mathematical struggles as "opportunities for delving more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions" (p. 48). Several researchers have worked to better understand those opportunities by studying school students' (e.g., Warshauer, 2015) and prospective teachers' (e.g., Zeybek, 2016) efforts to make sense of mathematics when engaging with challenging mathematical activities. Despite these gains, we lack research that reveals how and when learning occurs during the time that learners are engaged in a productive struggle. This information would help mathematics teacher educators to recognize and leverage their PTs' productive struggle and thus better support those PTs to be able to *support productive struggle in learning mathematics* (NCTM, 2014) in their future K-12 classes. Thus, this study explores the inner workings of PTs' productive struggle to investigate how and when those learning moments arise.

Literature Review

Productive struggle is a growing area of research that has encompassed three major themes: (a) types of struggles (e.g., DuCloux et al., 2018; Warshauer, 2015), (b) factors that foster productive struggle in classes (e.g., Roble, 2017; Russo et al., 2021), and (c) ways to help PTs to support productive struggle in their future classes (e.g., El-ahwal & Shahin, 2020; Warshauer et al., 2021). Since our work focuses on PTs' productive struggle, we elaborate on the first theme. Warshauer (2015) investigated 327 middle school students' struggles and developed a productive struggle framework that included four dimensions: (a) Task, (b) Student Struggle, (c) Teacher Response, and (d) Outcome. Within the Student Struggle dimension, she identified four kinds of student struggles: (a) *get started*, (b) *carry out a process*, (c) *give mathematical explanation*, and (d) *express misconception and errors* (Warshauer, 2015). These identified kinds of struggles have served as the foundation for other research in this theme. Sayster and Mhakure (2020) used Warshauer's kinds of student struggles to discuss 28 high school students' struggles with simplification of rational algebraic expressions. Their results aligned with Warshauer's kinds of student struggles. Other researchers have investigated PTs' struggles. Zeybek (2016) used Warshauer's kinds of student struggles to document the struggles of 48 pre-service middle grade teachers when engaging in non-routine tasks in a geometry course. Zeybek's findings aligned well with the results of Warshauer's study, thus she concluded that "struggle types are similar in all classrooms regardless of grade level or participants as long as participants are engaged in a high level task" (p. 411). DuCloux et al. (2018) used Warshauer's kinds of student struggles as a guide to analyze the perceptions of 32 prospective elementary, middle, and secondary teachers about their struggles. Their participants identified perseverance and struggling together with their peers as important aspects of their productive struggles. Although these studies provide us some information about PTs' productive struggles, they stop short of investigating the inner workings of productive struggle. Hence, our study aims to build on the existing research by exploring this research question: *What do PTs' productive struggles look like when they are engaged in a challenging mathematics task in a middle school mathematics methods course?*

Theoretical Framework

We ground our work in Warshauer's (2015) productive struggle framework mentioned above, with our adaptations shown in Figure 1. Rather than looking at tasks with a variety of cognitive demands, we used a *doing math* task (Box 1; Stein et al., 1996). Similarly, rather than looking at all possible outcomes, we looked only at interactions that led to a productive outcome (Box 4). To operationalize a productive outcome, we defined a *productive struggle* as an interaction that begins with some expression of struggle, verbal (e.g., "I'm confused", "I don't know", or long pause), or physical (e.g., deep sigh, putting hands in head) that provides the opportunity to better understand a well-specified statement of a mathematical truth—what we call a mathematical point (MP; Van Zoest et al., 2016)—and ends when some sense has been made of the MP and the focus of the interaction changes. Our work is situated in Boxes 2 and 3.

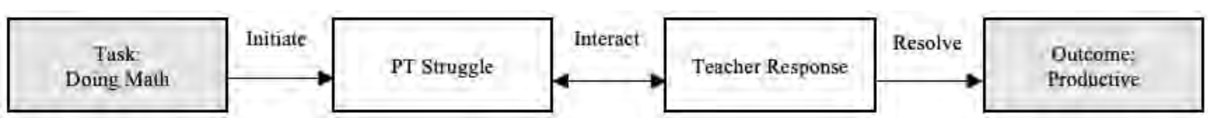

Figure 1: PT Productive Struggle Framework (Developed from Warshauer, 2015)

Methodology

This exploratory study is part of a larger ongoing project that investigates the teaching and learning that occurs in a middle school mathematics methods course (e.g., Ochieng, 2018; Van Zoest & Levin, 2021). The current study took place with the 18 PTs in the Fall 2021 course during the first two 90-minute class sessions. The PTs were engaged in the Frog Problem (see Figure 2), which met the criteria for a *doing math* task (Stein et al., 1996). (For more details about the Frog Problem, see Andrews (2000) and Dixon and Watkinson (1998).)

Figure 2: The Frog Problem Prompt and Representation of Two Frog Teams of Size Three

Data collection included videos and audio recordings of both class sessions, which were primarily whole-class discussion on the first day and small-group discussions on the second-day. We also gathered electronic copies of all PT written work and reflections from both days to help us better understand what a productive struggle looks like in detail. We first identified interactions that met our definition of productive struggle (see above). These interactions—

referred to as “productive struggles”—served as our primary data set for the study, with the other data sources referred to as needed to help us make sense of each productive struggle. Our analysis of these productive struggles was supported by coding the answers to these questions: (a) Who is struggling?, (b) What are they struggling with?, and (c) Why is it a productive struggle? In our broader analysis we looked at the role of the teacher response, but here we narrow the discussion of our results to what we found out about the Student Struggle dimension of Warshauer’s (2015) framework, identified as PT Struggle in Box 2 of Figure 1.

Results & Discussion

Based on our preliminary analyses, we hypothesize that a productive struggle consists of different types of struggles that can coexist simultaneously. We identified three types of struggles: *goal struggle*, *strategy struggle*, and *sub-strategy struggle*. We use an illustrative productive struggle (IPS) to elaborate on these types and to provide insight into the inner workings of PTs’ productive struggles and how they may differ from school students’ struggles (Warshauer, 2015). This particular productive struggle took place during a small-group discussion among four PTs (PT1, PT2, PT3, and PT4) that occurred on the second day of the classes’ engagement with the Frog Problem. At this point in the discussion, the group knew the actual numbers for frog teams of size 1, 2, 3, and 4. PT3 suggested an arithmetic sequence [$a_n = a_1 + (n - 1)d$] as a template for finding an equation that represents the fewest number of moves for any size team of frogs and the other PTs agreed. (Note that the difference of the fewest moves from each consecutive case is not constant, thus an arithmetic sequence does not apply to this situation.) The IPS began with the following exchange:

PT1: So wait, what would *d* be then?
PT3: That’s what I’m trying to figure out because the difference increases so [*d*]’s not consistent. So I don’t know that it’s technically arithmetic because the, an arithmetic sequence is consistent.
PT1: That’s true.
PT3: So I don’t know if I can change the *d* to be something like that.

This exchange initiated a productive struggle because it provided the PTs the opportunity to better understand this MP: *d is a constant that represents a common difference between each consecutive term in an arithmetic sequence.* The IPS ended approximately ten minutes later when the group made sense of that MP and agreed on a plan for a different approach (discussed in more detail below).

After the transcript excerpt above, the other PTs joined PT3 in her struggle by attempting to find *d* even though they recognized that the *d* in an arithmetic sequence needs to be constant. We noticed that their struggle was related to *express misconception and errors*, one kind of student struggle from Warshauer’s (2015) study. However, the result from our study is slightly different, since the PTs realized the misconception of using *d* as a common difference in arithmetic sequence when the differences in their data are not constant. We hypothesize that the PTs additional experience in learning mathematics may be responsible for their ability to recognize their misconception, leading to a more sophisticated version of this kind of struggle than that seen in Warshauer’s study with middle school students.

The PTs were struggling with making sense of *d* in an arithmetic sequence. However, while struggling with this specific concept, they also had been struggling with how to use an arithmetic sequence as a template to find the equation that represents their solution to the task. This is an example of what we call a *strategy struggle*. Hence, struggling with finding *d* became a *sub-*

strategy struggle since it represents small steps or elements in the proposed strategy. While engaged in both of these types of struggles, they also had been struggling with how to find an equation to represent the fewest number of moves for any size team of frogs. This struggle is defined as a *goal struggle* since the goal of this task is to generalize their obtained data to any size frog team. Since struggling with this sub-strategy struggle—making sense of *d* in an arithmetic sequence—brought an opportunity for the PTs to better understand the stated MP (see above), we hypothesize that with *doing math* tasks, resolving sub-struggle strategies may be where PTs' learning—their coming to better understand an MP—occurs. To describe more about how that learning occurs, we need to take into consideration context outside the IPS. From our broader analysis, the challenging mathematical task—the Frog Problem—initiated the PTs' goal struggle. Then, the PTs' goal struggle initiated the PTs' strategy struggle, followed by the PTs' sub-strategy struggle that led them to further their learning of the MP of this IPS. Thus although in this case there were three struggles going on simultaneously that were moving the PTs towards the goal of the lesson, it was the sub-strategy struggle that made this particular interaction productive in the sense of learning mathematics.

Towards the end of the 10-minute IPS, making a decision to change to a different focus after making sense of the *sub-strategy struggle* provided evidence that learning had occurred. Below is when the PTs realized they could not use the current approach and proposed a next strategy for the group to consider.

PT2: So instead of making it *d*, could almost like put a variable that was like the last variable I'm gonna add two then in place of d like that's what it would represent there?
PT1: So what do you mean?
PT2: Like, Ok so [PT3: I know what you're saying] Yeah, Like if you have like when you [PT3: But then we will have two variables] use the last one. Yeah that's the only thing is that there'd be a lot of variables.
PT4: Like and now we can do some substitution or elimination to figure out the other variable
PT1: So I feel like the total moves for the first four groups we can plug that in and maybe solve for a variable.
PT2: Yeah

Making these decisions as a group provided evidence that they were all engaged in making sense of the MP and had benefited from the learning opportunity.

Conclusion

The results of this exploratory study revealed the complexity of the inner workings of PTs' productive struggle. We hypothesize that a productive struggle can consist of different types of struggles—*goal struggle*, *strategy struggle*, and *sub-strategy struggle*—that may coexist simultaneously. These types of struggles expand our knowledge about struggle by adding detail about the nature of struggles to the already existing information about kinds of struggles (e.g., Warshauer, 2015; Zeybek, 2016). These findings highlight the importance of mathematics teacher educators recognizing the mathematical opportunities that may be present in their PTs' struggles and provide some insight into how mathematics teacher educators might leverage their PTs' productive struggles to better prepare those PTs to support their future students' productive struggles.

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